

**Flatness Based Control of Distributed Parameter  
Systems: Examples and Computer Exercises  
from Various Technological Domains**

Joachim Rudolph, Jan Winkler, Frank Woittennek

Dr. Joachim Rudolph, Dipl.-Ing. Frank Woittennek  
Insitut für Regelungs- und Steuerungstheorie  
Technische Universität Dresden  
Mommsenstraße 13  
D-01062 Dresden, Germany  
`{rudolph, woittennek}@erss11.et.tu-dresden.de`

Dipl.-Ing. Jan Winkler  
Institut für Kristallzüchtung  
Max-Born-Straße 2  
D-12489 Berlin, Germany  
`cwinkler@rcs.urz.tu-dresden.de`

Berichte aus der Steuerungs- und Regelungstechnik

**J. Rudolph  
J. Winkler  
F. Woittennek**

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## Abstract

Differential flatness is a concept which is very useful in the trajectory planning and feedback design for nonlinear finite dimensional systems, i.e., systems described by ordinary differential equations. The flatness based control methods place an emphasis on trajectory design and open-loop control. This aspect gains even more importance in infinite dimension, namely for distributed parameter systems with boundary control action, the mathematical models of which comprise partial differential equations, including the subclass of (linear and nonlinear) time delay systems.

The present booklet accompanies the lecture notes entitled “Flatness Based Control of Distributed Parameter Systems” written for a one-week course held at the Max Planck Institute for Dynamics of Complex Technical Systems at Magdeburg, Germany, in February 2003. These lecture notes put an emphasis on the generalization of the flatness property to distributed parameter systems and to its use in trajectory planning and open-loop control design. Time invariant linear systems with spatially distributed parameters and boundary controls are treated in a systematic manner. Basic ingredients of the method are operational calculus, series expansions, and integral representations. An extension to further classes of distributed parameter systems (nonlinear, time variant, in two space dimensions) is shown to be possible through a discussion of several examples.

The exercise booklet provides further examples, from various domains, allowing the interested reader to further study the material of the course, and to appraise its value in case studies of different complexity. In order to attain this aim, in addition to the exposition of questions and sketches of the answers also computer programs (written in MATLAB) are provided on an included CDROM.

The following examples are discussed:

- heat conduction in a Vertical-Gradient-Freeze crystal growth process,
- the tempering of crystals,
- piezoelectric benders,
- a seam welding process from packaging technology,
- horizontal transport of a water tank,
- signal transmission on an electric line modeled by the telegraph equation,
- and chemical reactors with recycle.



## Preface

The present booklet on examples and exercises accompanies the notes [Rud03] written for a one-week course at the recently founded “Max Planck Institute (MPI) for Dynamics of Complex Technical Systems”, Magdeburg (Germany) on February 24–28, 2003. This is the first institute of the Max-Planck-Society devoted to engineering sciences. It provides a highly interdisciplinary environment, where control engineers, biologists, mathematicians, chemists and chemical engineers closely cooperate.

The course is organized in cooperation with J. Raisch, the head of the “Lehrstuhl für Systemtheorie technischer Prozesse, Otto-von-Guericke Universität Magdeburg” and the “Systems and Control Theory Group, MPI Magdeburg”. The lecturers are

M. Fliess, Centre de Mathématiques et de Leurs Applications, ENS Cachan, and GAGE, Ecole Polytechnique;

H. Mounier, Centre de Robotique, Ecole des Mines de Paris;

P. Rouchon, Centre Automatique et Systèmes, Ecole des Mines de Paris,

and the authors of the present notes.

In the course flatness based design is discussed for several classes of infinite dimensional systems: linear distributed parameter systems with one dimensional space domain and lumped (mostly boundary) control action, some extensions to other classes (nonlinear or higher dimensional space domains, e.g.), as well as linear and nonlinear delay systems.

The exercises should allow the participants to deepen their understanding of the methods discussed in the lectures by doing calculations on case studies and simulations based on MATLAB code which can be found on the CDROM included. Examples from various technological domains are treated.

J. Rudolph, F. Woittennek  
Institut für Regelungs- und  
Steuerungstheorie  
TU Dresden

J. Winkler  
Institut für Kristallzüchtung Berlin,  
Institut für Regelungs- und  
Steuerungstheorie  
TU Dresden

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