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**Flatness Based Control of  
Distributed Parameter Systems:  
Examples and Computer Exercises  
from Various Technological Domains**

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# **Flatness Based Control of Distributed Parameter Systems: Examples and Computer Exercises from Various Technological Domains**

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## **Abstract**

Differential flatness is a concept which is very useful in the trajectory planning and feedback design for nonlinear finite dimensional systems, i.e., systems described by ordinary differential equations. The flatness based control methods place an emphasis on trajectory design and open-loop control. This aspect gains even more importance in infinite dimension, namely for distributed parameter systems with boundary control action, the mathematical models of which comprise partial differential equations, including the subclass of (linear and nonlinear) time delay systems.

The present booklet accompanies the lecture notes entitled “Flatness Based Control of Distributed Parameter Systems” written for a one-week course held at the Max Planck Institute for Dynamics of Complex Technical Systems at Magdeburg, Germany, in February 2003. These lecture notes put an emphasis on the generalization of the flatness property to distributed parameter systems and to its use in trajectory planning and open-loop control design. Time invariant linear systems with spatially distributed parameters and boundary controls are treated in a systematic manner. Basic ingredients of the method are operational calculus, series expansions, and integral representations. An extension to further classes of distributed parameter systems (nonlinear, time variant, in two space dimensions) is shown to be possible through a discussion of several examples.

The exercise booklet provides further examples, from various domains, allowing the interested reader to further study the material of the course, and to appraise its value in case studies of different complexity. In order to attain this aim, in addition to the exposition of questions and sketches of the answers also computer programs (written in MATLAB) are provided on an included CDROM.

The following examples are discussed:

- heat conduction in a Vertical-Gradient-Freeze crystal growth process,
- the tempering of crystals,
- piezoelectric benders,
- a seam welding process from packaging technology,
- horizontal transport of a water tank,
- signal transmission on an electric line modeled by the telegraph equation,
- and chemical reactors with recycle.