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A Branch-and-Price Framework for Workforce Scheduling Problems

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Due to the high cost of labor, the efficient utilization of employees is of paramount importance in industrialized countries. This is particularly true for service industries, which are usually characterized by high personnel expenses, varying demands for labor, and the fact that services are not suitable for storage. As employees cannot be hired or laid off easily in the short-term, employers require tools to determine efficient workforce schedules in order to gain/preserve a competitive advantage.

Ever since Dantzig[1] proposed the application of mathematical programming models and methods to the problem of scheduling toll collectors, there has been a sizeable number of publications on the topic of workforce scheduling. However, most of the earlier publications on this subject deal with rather simple models due to the inherent complexity of the underlying practical problem. Only during the last couple of decades and because of the recent abundance of computing power there has been a strong interest in advanced mathematical programming techniques in order to determine optimal or "good" solutions for practical problems that were formerly only tractable using simple heuristics. In particular, the airline industry has benefitted a great deal by numerous different applications of so-called branch-and-price algorithms, i.e., the hybridization of branch-and-bound and column generation techniques, to solve crew scheduling problems. Regarding more general workforce scheduling problems, however, there exist only few—and rather problem-specific—publications dealing with such algorithms.

The aim of this work is to present a flexible modeling framework that can be used for a large fraction of workforce scheduling problems arising in practice, particularly shift and days-off scheduling problems. Essentially, shift scheduling is concerned with the determination of a number of shifts that have to take place on each day of the underlying planning horizon so that the prescribed demand for labor is satisfied. Days-off scheduling denotes the process of sequencing these shifts and assigning the resulting sequences to employees. The framework proposed in this work is based on a network representation, which permits a human scheduler to easily alter and augment an existing model. To the best of our knowledge, a similar unified approach has so far not been proposed in literature.

In order to determine an optimal or—depending on the size of the problem—a good feasible solution for the workforce scheduling problem under consideration, a number of branch-and-price algorithms is proposed. Branch-and-price denotes a sophisticated class of algorithms which descends from the well-known class of branch-and-bound algorithms. In the course of branch-and-price, we iteratively solve a linear relaxation of the underlying integer program using the column generation principle. The underlying idea is to consider only a small subset of all decision variables and use implicit, problem-specific knowledge to dynamically generate new variables with negative reduced costs. For the workforce scheduling problems considered in our work, this so-called subproblem of column generation amounts to the determination of a set of resource-constrained shortest paths in a network. There exist only a very limited number of publications dealing with branch-and-price methods for days-off scheduling problems, and no such algorithm has yet been applied to shift scheduling problems. The algorithms presented in this work moreover exhibit some novel features which have not yet been examined in similar applications.

Given the complex nature of the mathematical programming techniques involved, our branch-and-price algorithms offer themselves particularly to the solution of small to medium-sized practical problems that are characterized by highly volatile labor demands, a heterogeneous workforce where the utilization of these employees is considered a major cost driver, and a heterogeneous set of labor tasks that are eventually aggregated to shifts and shift sequences.

[1] GB Dantzig (1954): A Comment on Edie's "Traffic Delays at Toll Booths". *Operations Research* 2:339–341.