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Zusammenfassung:  
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## Residuated Structures with Involution

The main structures which will be considered in the PhD thesis "Residuated Structures with Involution" by Annika Meike Wille are involutive residuated lattices. From the algebraic point of view they provide a framework for various structures such as Boolean algebras, lattice-ordered groups and MV-algebras. If we look at them from a logical perspective, they are the algebraic counterparts of non-commutative linear logic without exponentials. The aim of the dissertation is to learn more about decidability problems, finite axiomatizability, and about the subvariety lattice of the variety of involutive residuated lattices.

In the thesis the attention is turned to residuated lattices with a distinguished element which gives rise to an involution on the lattice reduct. On the logical side this will give us negation, on the algebraic side we receive among other things a residuated lattice which has an involutory order anti-automorphism. On part of the dissertation concerns the variety of pointed residuated lattices and more specifically its subvarieties of weakly involutive residuated lattice and involutive residuated lattices. It will be shown that the varieties of Boolean algebras, lattice-ordered groups, bounded generalized MV-algebras, MV-algebras and certain reducts of relation algebras can be seen as subvarieties of the variety of weakly involutive residuated lattices.

Besides other results a cut-free Gentzen system for involutive residuated lattices is presented and an algebraic proof of completeness is provided. In particular, this proves that the equational theory of involutive residuated lattices is decidable. In the last chapter the undecidability of the word problem will be shown for various residuated lattice varieties and their classes of finite members including residuated lattices, pointed residuated lattices, weakly involutive residuated lattices, and involutive residuated lattices. This implies that the quasi-equational theory of all these varieties is undecidable.