Steuerungs- und Regelungstechnik

Joachim Rudolph

Flatness-Based Control

An Introduction

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FLATNESS-BASED CONTROL
An Introduction

Joachim Rudolph

Notes for a course

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