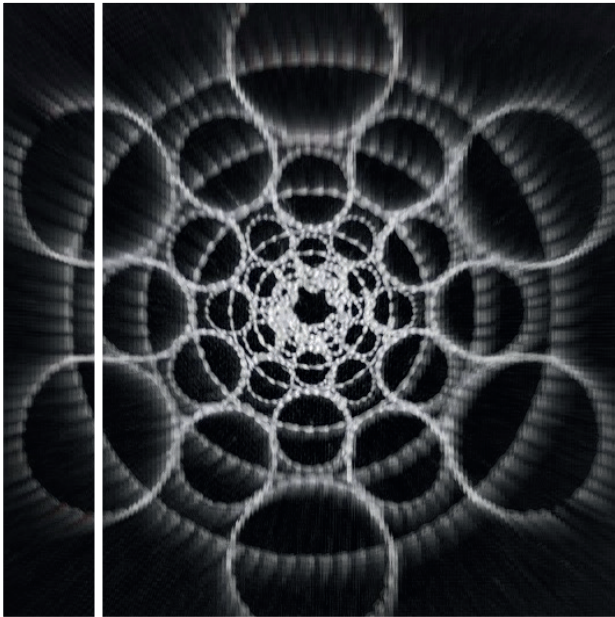


# **ADVANCED METHODS FOR DIGITAL SYSTEMS AND SIGNAL PROCESSING**

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# Preface

Recently, paging through the material, strewn over our papers in scientific journals, somebody of us has remarked that it would be great to collect the main outcome at the end of our scientific career, and after pre-processing to publish as a monograph. By the same token, we have taken some proficiency by preparing our prior monographs: "Advanced methods for short signal spectrum estimation" and "Reinforced methods for dynamical system identification and adaptive control", published by Shaker in 2016 and 2017, respectively. The previous scientific outcomes, that are not included in the abovementioned monographs and the recently got issues, are rolled today into the set of new books continuing the previous ones.

The objective of our exploration is models and approaches of the analysis and calculation of their main characteristics, filtering and parametrical identification of adaptive filters and systems, proposed for self-organized control. There is a need at the very beginning, to find the factor that will be common for the abovementioned filters and/or systems. Such a characteristic could be the ordinary impulse response of the linear filter or system, not only due of the relevance itself, but also much more because of its tight association with the well-known convolution operation that is substantial in the digital signal processing. It is necessary to analyze the calculation aspects of linear convolution in order to establish the common features and their differences for distinct models to be applied. Such an approach would allow us to concentrate in one unit the relations among the matrix block impulse response, block impulse response, matrix impulse response and that of LTV systems, frozen-time LTV systems, LPTV systems, LTI systems, and nonlinear TIBO systems.

Just like the preceding creations, this one is the result of approximately thirty-years experience of the authors in the research on adaptive filters and systems as well on their identification for adaptive control. The roots of that started at the former Laboratory of Adaptive systems of the Institute of Physical and Technical Problems of Energetics in Kaunas, where we had acquired useful skills. The experience in scientific research we have acquired

in the Group of Control of Technological Processes at the Process Recognition Department of the Mathematics and Informatics Institute in Vilnius. Teaching of Informatics at the Vilnius Educational Science University in the Department of Informatics, and the Digital Signal Processing at the Department of Electronic systems of the Electronics Faculty of Vilnius Gediminas Tech University stimulated uprise of our experience. Collaborative investigation skills have been sharpened during the development and execution of the Deutsche Forschungsgemeinschaft project "Echtzeit-Optimierung den Grössen Systemen" (chief of the project Prof. P.J. Huber) as well as the New Visby project Ref No. 2473/2002 (381/T81) of the Royal Swedish Academy of Sciences and the Swedish Institute (chief of the project Prof. D.Navakauskas), during the long-time visit at the Division of Automatic Control of Linköping University (head of the control group Prof. L.Ljung).

The monograph is aimed at the three major groups of readers: senior undergraduate students, graduate students, and scientific research workers in electrical engineering, computer engineering, computer science, and digital control. The monograph could be suitable for researchers as the background reading and references, as well.

The monograph consists of 11 Chapters and Appendixes A, B, and C.

The text begins with an introduction to the authors' creation, where the reasons for development of monograph are shown and explained briefly. Here the basic approaches to the problems to be analyzed in the book are outlined and expressed in short. Various but, of course, not all the references to the well-known scientific works are presented.

In Chapter 2, the common relationship between scalar and BIRs of LTV, frozen-time LTV, LPTV and LTI systems has been explored theoretically.

In Chapter 3, a simple and computationally efficient method for computation of the transfer function and other characteristics of MSs is explored.

Chapter 4 contains measures to describe the matrix impulse response sensitivity of state-space MV systems with respect to parameter perturbations. The parameter sensitivity is defined as an integral measure of the matrix impulse response in respect of the coefficients. A state-space approach is used to find a realization of impulse response that minimizes the sensitivity measure.

Chapter 5 presents the input-output relationship of the PTV systems, impulse response of the PTV state-space system, and the transfer function of the PTV system. The coefficient sensitivity is investigated by using a PTV state-space system in which periodically time-varying coefficients are varied stochastically.

In Chapter 6, the idea of converting the LPTV filter to a block time-invariant filter is used for determining the inversion of LPTV filters. Controllability, observability, and stability of the inversion of LPTV filters are analyzed.

In Chapter 7, using the expansion of the 2-D inverse operator into a power series, the formulas are derived for calculating the approximated inverse filter and signal restoration in the time and frequency domains.

Chapter 8 considers the development of optimal and tuned models and ordinary well-known on-line parametrical identification procedures for an IS, using the current observations. Such IS models are obtained in the case of correlated additive noise corrupting the output of the initial system.

Chapter 9 analyzed a fast convolution procedure for a nonstationary system. The objective of the chapter is to update the fast convolution on-line over the sensor network without redundant CMADs operations, where the previous samples in respective matrices are partly replaced.

Chapter 10 introduces the 2-D convolution in the on-line mode, using the approach based on Horner's method.

In Chapter 11, the approach is developed for robust recursive state estimation of LTI dynamic systems, described in a state-space, in the case of additive noise with time-varying outliers. It is realized by means of the bank of the robust Kalman filters, working in parallel in the presence of time-varying outliers in a noisy frame.

In Appendix A, the convolution algorithm is modified according to Horner's method. It is realized in a pipelined-parallel mode in order to work effectively in real-time applications.

In Appendix B, the example with the convolution recursive calculation results is given for a system having non-causal impulse response.

Appendix C represents outlier tests, proposed by Grubb's (1960, 1968) and further by Burke (1998). They are applied for outlying data search on-line in the calculation.

Chapters 2 – 7 are written by K.Kazlauskas, while Chapters 8 – 11 and Appendices A – C are written by R. Pupekis. Chapter 1 is written jointly.

The authors thank the Institute of Data Science and Digital Technologies of Vilnius University as well the Faculty of Mathematics and Informatics of Vilnius University for a long-term financial support of their scientific researches.

Reviewers of the Monograph: Prof. dr. Vytautas Slivinskas, Director of the joint Lithuanian and Canadian firm 'Asteria', and dr. Gintautas Tamulevičius, Associate Professor of the Vilnius Gediminas Tech University.

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06 2021

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# Acronyms

APES	amplitude and phase estimation
AFR	amplitude frequency response
AR	autoregressive
ARMA	autoregressive moving average
BIR	block impulse response
CMAD	complex multiplication and addition
DFT	discrete Fourier transform
DFFT	discrete fast Fourier transform
DS	direct system
DSP	digital signal processing
FFT	fast Fourier transform
FIR	finite impulse response
FR	frequency response
f.s.	frequency samples
IDFT	inverse discrete Fourier transform
IF	instantaneous frequency
IPM	inverse polynomial matrix
IS	inverse system
LP	linear prediction
LS	least squares
LTI	linear time-invariant
LPTV	linear periodically time-varying
LTV	linear time-varying
MA	moving average
MARMA	multivariate autoregressive moving-average
MAR	multivariate autoregressive
MIMO	multi-input multi-output
MS	multivariate system
PhFR	phase-frequency response
PM	polynomial matrix
PTV	periodically time-varying



RLS	recursive least squares
RGLS	recursive generalized least squares
RML	recursive maximum likelihood
SIR	scalar impulse response
SISO	single-input single-output
SNR	signal-to-noise ratio
2-D	two-dimensional
TIBO	time-invariant block-oriented

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