

FLATNESS BASED CONTROL OF DISTRIBUTED PARAMETER SYSTEMS

Joachim Rudolph

Notes for a course
at the Max Planck Institute for Dynamics of Complex Technical Systems
at Magdeburg, Germany, on February 24-28, 2003

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Abstract

Ever growing performance requirements as well as new technologies require an increasing number of control systems being designed on the basis of mathematical models comprising partial differential equations or time delays. These classes of models, and control methods adapted to them, may be expected to play an important role in high-technology applications in the next few years, similar to what happened for nonlinear systems and nonlinear control in the last decade.

The notion of differential flatness of nonlinear finite dimensional systems, described by ordinary differential equations, has given rise to various powerful methods for motion planning and control design. It plays an increasing role in industrial applications of nonlinear control. Flatness based control methods place an emphasis on trajectory design and open-loop control. Unfortunately, this aspect has not always attracted the consideration it requires both in the control theoretic literature and in control education.

The careful design of feed-forward, or steering, control gains even more importance in infinite dimension, namely for distributed parameter systems with boundary control action, the mathematical models of which comprise partial differential equations, and also for the subclass of (linear and nonlinear) time delay systems.

As a consequence, the flatness based approach has been generalized to the infinite dimensional case. Parameterizing the system trajectories by a so-called flat output, for many infinite dimensional systems efficient motion planning and open-loop (feed-forward) control design can now be achieved in a way similar to the one followed with nonlinear flat systems. The feedback linearization and eigenvalue assignment methods known from nonlinear finite dimensional systems have also been shown to generalize to delay systems.

The emphasis of the present notes is put on the generalization of the flatness property to distributed parameter systems and to its use in trajectory planning and open-loop control design. Time invariant linear systems with spatially distributed parameters and boundary controls are treated in a systematic manner. Basic ingredients of the method are operational calculus, series expansions, and integral representations. An extension to further classes of distributed parameter systems (nonlinear, time invariant, in three space dimensions) is shown to be possible through a discussion of several examples.

Before dealing with distributed parameter systems, the flatness based approach to finite dimensional nonlinear systems is briefly recalled and its generalization to linear and nonlinear systems with (constant) time delays is outlined, too.

A considerable number of examples illustrates the use of the methods proposed.

Preface

The present notes have been written for a one-week course entitled “Flatness based control of distributed parameter systems” to be held at the recently founded “Max Planck Institute (MPI) for Dynamics of Complex Technical Systems” at Magdeburg, Germany on February 24–28, 2003. It is the first institute of the Max-Planck-Society devoted to engineering sciences and provides a highly interdisciplinary environment, where control engineers, biologists, mathematicians, chemists and chemical engineers closely cooperate.

The course is organized in cooperation with J. Raisch, the head of the “Lehrstuhl für Systemtheorie technischer Prozesse, Otto-von-Guericke Universität Magdeburg” and the “Systems and Control Theory Group, MPI Magdeburg”. Thanks also to financial support by the MPI, we could gain the following lecturers for that course:

M. Fliess, Centre de Mathématique et de Leurs Applications, ENS Cachan, and GAGE, Ecole Polytechnique;

H. Mounier, Centre de Robotique, Ecole des Mines de Paris;

P. Rouchon, Centre Automatique et Systèmes, Ecole des Mines de Paris;

J. Winkler, Institut für Kristallzüchtung, Berlin, and Institut für Regelungs- und Steuerungstheorie, Technische Universität Dresden;

F. Woittennek, Institut für Regelungs- und Steuerungstheorie, Technische Universität Dresden,

the list being completed by myself.

The aim of the one-week course is to provide a thorough understanding of the flatness based design for several classes of infinite dimensional systems: linear distributed parameter systems with one dimensional space domain and lumped (mostly boundary) control action, some extensions to other classes (nonlinear or higher dimensional space domains, e.g.), as well as linear and nonlinear delay systems. The course will comprise both lectures and computer exercises.

These exercises should allow the participants to deepen their understanding of the methods discussed in the lectures by doing calculations on case studies and simulations based on MATLAB/Simulink. Examples from various technological domains will be treated. Another booklet [RWW03] has been written as a support of these exercises.

Instead of handing out a variety of certainly most interesting separate contributions from each of the lecturers I felt that it might be useful to provide a single (relatively) concise and coherent text. It largely corresponds to (the more applied) parts of a treatise that I have submitted as a habilitation thesis (in German) to the “Fakultät Elektrotechnik und Informationstechnik” at TU Dresden in January

2002, and which should be published this year. Of course, the present notes are exhaustive neither w.r.t. the wide subject nor w.r.t. the contents of the lectures given at the course. Hopefully, it will, nevertheless, serve as a useful introduction and survey, and a few readers will be led to contribute to the development of further flatness based methods and apply these methods to the control problems they meet in their practical work.

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Dresden, January 2003

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He would also like to express his thanks to Prof. Dr.-Ing. Dr. rer. nat. K. Reinschke, the director of the “Institut für Regelungs- und Steuerungstheorie” at TU Dresden, where he could find most favorable working conditions, for his interest and support.

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