

# **Reliable Modelling of Electromechanical Systems using Macromodell Approach for Control Purposes**

**Von der Fakultät für Elektrotechnik und  
Informationstechnik**

**der Universität der Bundeswehr München**

**zur Erlangung des akademischen Grades eines**

**Doktor-Ingenieur**

**(Dr.-Ing.)**

**genehmigte Dissertation**

**von**

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*der Bundeswehr*  
**Universität**  **München**

Neubiberg  
2011

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Tag der Promotion: 19.10.2011

Forschungsberichte Elektrische Antriebstechnik und Aktorik

Band 10

**Nizar Khateeb**

**Reliable Modelling of Electromechanical Systems  
using Macromodell Approach for Control Purposes**

Shaker Verlag  
Aachen 2011

**Bibliographic information published by the Deutsche Nationalbibliothek**

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Zugl.: München, Univ. der Bundeswehr, Diss., 2011

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Printed in Germany.

ISBN 978-3-8440-0593-6

ISSN 1863-0707

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

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*To my parents*



## Abstract

The macromodelling, the theme of this thesis, is a new modelling approach for the electromechanical systems. This approach depends on describing the studied system using some high reduced data defined the system behaviour. The key point is the use of the finite element analysis to generate this data required to accomplish high accuracy modelling methodology. Due to this scope, the macromodelling presents a technique to replace the large dimension discrete data, stored in look-up tables, with an analytical formulation. On the other hand, the macromodelling approaches serves as a topology to link two simulation environments: The finite element method tool (with its high accuracy) and the system simulator (with its high flexibility).

Generally, the macromodelling procedure can be divided into two tasks: Reducing the amount of the data required to describe the studied system and finding the appropriate analytical formulation of the reduced data.

To perform the new methodology, linear algebra is pointed as a mathematical framework to perform the macromodelling algorithms. Otherwise, the energy function method covers the physical framework which is used to derive the main system equations.





## Acknowledgments

I would like to express my gratitude to my advisor Prof. Dieter Gerling. I am deeply indebted to him for many reasons: he has provided encouragement and technical support; he has opened up many academic and personal opportunities. I would also like to thank Dr. Hans-Joachim Köbler, Dr. Harald Hofmann and Klaus Mühlbauer for their generously help during my working in the institute for electrical drives and actuators. A special thanks to Dr. Benno Lange, Dr. Gurakuq Dajaku and Rainer Hildebrand who have been a constant friends.

Throughout my life, I have been influenced by many role-modells. I would like to especially acknowledge my parents. They have encouraged me to pursue my interests, and have been both accepting and supportive of my decisions. I am indebted to them for many things, but essentially for their example of what it means to give without waiting for taking.

Most importantly, I would like to give my special thanks to my wife and my children for all support and encouragement. They have been a source of inspiration and this work is a tribute to them. I greatly appreciate their patience and understanding in my time of need.

*Nizar Khateeb*

*May 2010*



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## List of Notation

### General Acronyms

AnROM	Analytic ROM
BF	Basis Functions
CAnROM	Concentrated ROM
CV	Curve Fitting
FEA	Finite Element Analysis
FEM	Finite Element Method
MOR	Modell order Reduction (equivalent to macromodell and ROM in literature)
OP-ROM	Orthogonal Projection based ROM
PM	Permanent magnet Motor
ROM	Reduced Order Modell (100 % equivalent to Macromodell)
SRM	Switched Reluctance Motor
SVD	Singular Value Decomposition
TTRR	Three term recurrence relation

### Symbols and Definitions

**General notice:** When a letter use to describe two signals (or variables): physical and mathematical then the physical signal is recognized with the sign  $[ \tilde{ } ]$  upon it, e.g.,  $\tilde{u}$  is a physical signal (source voltage), where  $u$  is a basis function.  $\tilde{R}$  and  $\tilde{i}$  are also Resistance and current respectively.

$A$	Matrix contains the coefficients of the Fourier series resulted from curve fitting
$a_0, a_i$	Fourier coefficients
$B$	Coefficient of friction
$b_i$	Fourier coefficients
$c$	The coefficients matrix of the spectral decomposition
$c_r$	Macromodell matrix (reduced order modelled $c$ )

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$c_{ij}$	Coefficients of the macromodell matrix $c_r$ or the coefficients matrix $c$
$C$	Matrix of the concentrated polynomials $C_0(i), C_1(i) \dots$
$dW_e$	The differential change in the electrical energy
$dW_f$	The differential change in the magnetic field energy
$dW_m$	The differential change in the mechanical energy
$e$	Intermediate macromodell
$E$	Back emf factor
$f(x)$	Mathematical modell of 1-Dimension data (1 variable function space)
$f(x,z)$	Mathematical modell of 2-Dimension data (2 variable function space)
$f$	Vector contains the function values $f(x_k)$ or matrix contains the function values $f(x,z)$
$f_r$	Approximated function vector or matrix by reducing the number of the basis function.
$\mathbb{F}$	Studied function space.
$FI$	Basis functions matrix of the co-energy –current look-up table
$FI_r$	Reduced basis function matrix of the co-energy –current look-up table
$FIo$	Orthonormal basis function matrix the co-energy –current look-up table
$FIt$	Upper triangular matrix of the co-energy –current look-up table
$FX$	Basis functions matrix of the co-energy –angle look-up table
$FX_r$	Reduced basis function matrix of the co-energy –angle look-up table
$FXo$	Orthonormal basis function matrix the co-energy –angle look-up table
$FXt$	Upper triangular matrix of the co-energy –angle look-up table
$F_{el}$	Electromagnetic force
$F_r$	Friction coefficient
$\tilde{i}$	Current variable or current vector
$i$	Index of a coefficient in a matrix or vector
$I$	Identity matrix
$j$	Index of a coefficient in a matrix or vector
$J$	Concentrated macromodell
$\tilde{J}$	Moment of inertia.
$k$	Index of a coefficient in a matrix or vector
$l$	Index of a coefficient in a matrix or vector
$L$	Inductance
$m$	Constant refers to the dimension of vectors in the columns space of $\mathbb{F}$
$n$	Constant refers to the dimension of vectors in the row space of $\mathbb{F}$
$p$	Reduced numbers of the basis functions of the columns space

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$P$	Matrix contains the coefficients of the fitting polynomials
$q$	Reduced numbers of the basis functions of the row space
$Q$	Orthogonal matrix (in QR orthonormalization)
$R$	Upper triangular matrix (in QR orthonormalization)
$\tilde{R}$	Phase winding resistance
$S$	A diagonal matrix containing the singular values of studied matrix
$S_r$	A diagonal matrix with the $r$ nonzero singular of studied matrix
$t$	Time variable
$T$	Electromagnetic torque
$T_p$	Period of a function
$T_l$	Load torque
$T$	Mechanical angle [ $^\circ$ ]
$T_{mech}$	
$T_{el}$	Electrical angle [rad]
$\tilde{u}$	Source voltage
$u_j$	Basis function of function space
$u_{ind}$	The induced voltage
$U$	Basis functions matrix ( $U$ in SVD is a orthonormal matrix)
$U_r$	Reduced basis functions matrix
$U_o$	Orthogonal matrix from $U_r$
$U_t$	Upper triangular matrix from $U_r$
$v_i$	Basis function of function space (with 2 or more dimensions)
$V$	Basis functions matrix ( $V$ in SVD is a orthonormal matrix)
$V_r$	Reduced basis functions matrix
$V_o$	Orthogonal matrix from $V_r$
$V_t$	Upper triangular matrix from $V_r$
$Wco_{FEM}$	FEM simulated co-energy look-up table
$Wco_{ROM}$	Reduced order modelled (macromodelled) co-energy look-up table
$WI(i)$	Co-energy as function of the current
$WI_r$	Reduced co-energy -current function matrix
$WX(\alpha)$	Co-energy as function of angle
$WX_r$	Reduced co-energy -angle function matrix
$\tilde{WIX}(i, \alpha)$	Co-energy as function of both angle and current

$Wco_{AnROM}$	Analytic reduced order modelled (macromodelled) co-energy look-up table
$Wco_{CAnROM}$	Concentrated analytic reduced order modelled (macromodelled) co-energy look-up table
$x$	A general variable of a function $f$ or vector in the 1-dimension function space $\mathbb{F}$
$Y$	The studied look-up table (measured or FEM-simulated data) whose dimension is $m \times n$ matrix
$z$	A second variable or vector in the 2 or more dimensions function space $\mathbb{F}$

### Greek letters

$\alpha$	Rotor position or a vector of 1-dimension angle values
$\psi$	Flux linkage (in literature also $\lambda$ )
$\omega$	Angular speed

### Algorithm Names and Acronyms

[TTRR_Poly]	The Gram-Schmidt orthogonalization based three term recurrence relation
[Poly_Norm]	Orthonormalization the TTRR-generated polynomials
[Coe_Orth_Poly]	Computing the coefficients of the orthogonal polynomials
[Coe_Orth_Norm_Poly]	The coefficients of the orthonormal polynomials