Archimedean Approximations

Uwe Kraeft

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Berichte aus der Mathematik

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Preface

This book gives a short and basic treatment of Archimedean Approximations. These are convergent sequences of terms in which one or more variables can be made so small as you want.

In chronological order with former books on Pythagorean Triples, Euclidean Sequences, and Diophantine Equations this fourth text is written for all who are interested in basic mathematics and have an elementary knowledge of analysis. Especially it may be of interest for the student who begins to study the elements of number theory and history of mathematics. The book is dedicated to Archimedes because he was a new type of mathematical scientist and engineer and his famous books show the approximations which are also nowadays used in differential and integral calculus. His works, which are based on those of older philosophers and mathematicians, are outstanding and contain a great part of own contributions. Fortunately some of his Books were copied many centuries and so preserved until now.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in January 2001 Uwe Kraeft

Symbols

$\Rightarrow$	by this follows
$\in$	is element of (is contained in)
$\notin$	is no element of (isn't contained in)
$\subset$	is subset of (all elements are contained in)
$\cup, \cap$	union and intersection of sets
$\varnothing$	the empty set
{a,b,c}	an example of a set with elements a, b, and c
{{a,a,c}}	an example of an assemblage with individuals a, a, and c
a, A, α ...	in this text mainly natural numbers 1, 2, 3, ...
$\underline{v}$	vector
N	set of natural numbers 1, 2, 3, ...
N^0	$N \cup \{0\}$
P^1	$P \cup \{1\}$ primes P included 1
{-N}	$=\{-n; n \in N\}$ set of negative integers -1, -2, -3, ...
Z	$=N \cup \{-N\} \cup \{0\}$ set of integers
Q^+	set of positive rational numbers a/b, $N \subset Q^+$
R	set of real number algorithms
C^1	set of complex numbers a+bi
C^{1+n}	set of hypercomplex numbers
$\underline{v}$	vector
=	equal (not between rational and irrational numbers)
$\cong$	so near as you want but not identical
$\approx$	approximately, rounded
$\neq$	not equal
<, >	less, greater
(a,b,c)=1	greatest common divisor (factor) gcd$\in$N of a,b,c is 1
$\sum_{i=1}^{n} a_i$	$=a_1+a_2+ \ldots +a_n$
n!	1*2*3*...*n
$f'(x)=\frac{dy}{dx}$	first derivative of f(x) with dx$\neq$0
F(x)	integral of f(x): $\int_a^b f(x)dx$ with dx$\neq$0

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