Bernoulli, Euler, Stirling, Figurate Numbers and Factorials

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Preface

The Jakob Bernoulli Numbers or Bernoulli Numbers BN, Euler Numbers EN, Stirling Numbers SN, and figurate numbers are known since centuries and have got many applications. They are originally the result of finite rational sums which are deduced by binomial coefficients and often given by factorials, which show the laws of prime formation. On the other side, definitions and recurrent formulae with irrational results are existing for these numbers. Especially the BN and factorials are of greatest importance in number theory and lots of theorems and conjectures have been published. Therefore, they are also today an object of research. The book can show only a small choice of what is thought to be most important.

This text is part of a sequence of studies in number theory. As in former contributions, the author has tried to find and cite the relevant literature so far as possible. The proofs are made or modified by the author; if literature was used, a notice is given. If there is no such hint, there is certainly no guarantee that similar proofs haven't been published by other authors before.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in April 2006 Uwe Kraeft

Choice of symbols

$\Rightarrow, \Leftarrow, \Leftrightarrow$	by this follows (in the given directions)
$\in$	is element of (is contained in)
$\cup$	union
$A=\{a,b,c\}$	an example of a set A with elements a, b, and c
N	set of natural numbers 1, 2, 3, ...
P	primes of N 2, 3, 5, ... (P^1 with the unity)
N^0	$N\cup\{0\}$
N^-	$\{-n;\ n\in N\}$, set of negative integers -1, -2, -3, ...
Z	$=N\cup\{N^-\}\cup\{0\}$, set of integers
Q	set of rational numbers a/b with $a\in Z$, $b\in N$
R	set of real number algorithms
$\binom{a}{b}$	$\frac{a!}{b!(a-b)!}=\frac{a(a-1)(a-2)*...*(a-b+1)}{b(b-1)(b-2)*...*1}$ binomial coefficient
$(x)^k$; $(x)_n$	$\frac{(x+k-1)!}{(x-1)!}$ rising factorial; $\frac{x!}{(x-n)!}$ falling factorial
$\left[\begin{matrix}n\\k\end{matrix}\right]$	s(n,k) Sterling Number of the first kind (see chapter 6)
$s^+(n,k)$	$\lvert s(n,k)\rvert$
$\left\{\begin{matrix}n\\k\end{matrix}\right\}$	S(n,k) Sterling Number of the second kind (see chapter 6)
B_n	Bernoulli Numbers if type is clear (see chapter 2)
B^0_n	positive and negative Bernoulli Numbers with $B^0_0=1$, $B^0_1=-1/2$, $B^0_{2n}=(-1)^{n-1}B^+_n$, and $B^0_{2n+1}=0$
$B^+_n=B^*_n$	positive Bernoulli Numbers $\neq 0$
$B^{\pm}_n$	positive and negative Bernoulli Numbers $\neq 0$
β_n	tangent numbers or coefficients (see chapter 2)
E_n	Euler Numbers if type is clear (see chapter 2)
E^0_n	positive and negative Euler Numbers with $E^0_0=1$, $E^0_{2n}=(-1)^nE^+_n$, and $E^0_{2n-1}=0$
$E^+_n=E^*_n$	positive Euler Numbers $\neq 0$
$E^{\pm}_n$	positive and negative Euler Numbers $\neq 0$
f_n	factorial numbers (see chapter 8)
$=$	equal (identical) by axioms or definitions
$\cong$	so near as you want but not identical
$\approx$	about, rounded, can f.e. be approximated for great n
$\equiv$	$a\equiv b \pmod c \Leftrightarrow a\equiv b_c \Leftrightarrow (a-b)/c\in Z$ for $a,b\in Z$, $c\in N$

BN	Bernoulli Number
EN	Euler Number
SN	Stirling Number
PT	Pythagorean Triple
FLT	Fermat's Last Theorem
f.e.	for example (e.g.)

The order of this sequence of texts on number theory is twofold. The order following the date of printing is given at the end of this book. Another grouping is got by the colours of the covers after disciplines as follows:

arithmetic number theory:	light blue
sequences and series:	dark green
Diophantine Equations:	orange
algebraic number theory:	dark red
topological number theory:	purple
analytic number theory:	dark blue
statistical number theory:	light green
special numbers:	dark yellow
textbooks:	light yellow

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