

A Short History of Number Theory

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2005

Berichte aus der Mathematik

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Shaker Verlag
Aachen 2005

Bibliographic information published by Die Deutsche Bibliothek

Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the internet at <http://dnb.ddb.de>.

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Printed in Germany.

ISBN 3-8322-4496-4

ISSN 0945-0882

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

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Preface

Mathematics and number theory have their roots in the darkness of history. At least the last 10000 years have seen faster and faster growing developments, which were connected in the beginning with practical requirements. There was a maximum in the golden time of number theory from the 18th until the end of the 19th century. Also in our days, there are found important new ideas and proofs. As a result, there is a vast literature, which an interested person cannot read in his or her life.

This book is only a short summary, which gives a personal view of the author. The history is shown by few chosen examples. The names are normally given in the order of the birthday, generally without personal data, and are representing not only the works of a person but especially the knowledge of that time, school, or university. In most cases, the roots of developments can be seen long before the publication of a conjecture or a proof were available. Examples of older results from the beginning of the disciplines are discussed usually longer than the well known new methods of our days because the former may be also today of interest. The overview is no textbook of number theory but it should enable the reader to find the most important works. The used terminology could only be explained in few cases. The focus is in the beginning all of mathematics and geometry because the principal difficulties, which were seen in former times, are a matter of number theory. The definition of the latter is here very wide and comprises all what is important for understanding, characteristics, calculations, results, and theorems in using numbers (natural numbers, integers, rational numbers, real number symbols, complex numbers, and new constructions following chosen axioms).

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in September 2005

Uwe Kraeft

Choice of symbols

$\Rightarrow, \Leftarrow, \Leftrightarrow$	by this follows (in the given directions)
\forall	for all
\exists	there is/are
\in	is element of (is contained in)
\cap	intersection
$A=\{a,b,c\}$	an example of a set A with elements a, b, and c
$-a, b^{-1}, c'$	inverse elements
\mathbb{N}	set of natural numbers 1, 2, 3, ...
\mathbb{P}	primes of \mathbb{N} 2, 3, 5, ...
\mathbb{N}^0	$\mathbb{N} \cup \{0\}$
\mathbb{N}^-	$\{-n; n \in \mathbb{N}\}$, set of negative integers -1, -2, -3, ...
\mathbb{Z}	$=\mathbb{N} \cup \{\mathbb{N}^-\} \cup \{0\}$, set of integers
\mathbb{Q}	set of rational numbers a/b with $a \in \mathbb{Z}, b \in \mathbb{N}$
\mathbb{R}	set of real number algorithms
$\mathbb{Q}(\mathbb{R})$	\mathbb{Q} or \mathbb{R}
$\mathbb{Q}(I)$	adjunction of I to \mathbb{Q}
$=$	equal (identical) by axioms or definitions
\cong	so near as you want but not identical
\approx	about, rounded, can f.e. be approximated for great n
\equiv	$a \equiv b \pmod{c} \Leftrightarrow a \equiv b_c \Leftrightarrow (a-b)/c \in \mathbb{Z}$ for $a, b \in \mathbb{Z}, c \in \mathbb{N}$
PT	Pythagorean Triple
FLT	Fermat's Last Theorem
gcd	greatest common divisor
f.e.	for example (e.g.)

Choice of mathematicians who gave contributions to number theory
after www-groups.dcs.st-and.ac.uk/~history/Indexes

about BC

1680-1620	Ahmes
624 - 546	Thales
580 - 520	Pythagoras
490 - 430	Zeno of Elea
470 - 410	Hippocrates
428 - 350	Archytas
428 - 347	Plato
415 - 369	Theaetetus
408 - 355	Eudoxus
384 - 322	Aristotle
325 - 265	Euclid
287 - 212	Archimedes
276 - 197	Eratosthenes
262 - 190	Apollonius

about AD

10 - 75	Heron
60 - 120	Nicomachus
85 - 165	Ptolemy
200 - 284	Diophantus
290 - 350	Pappus
335 - 395	Theon
370 - 415	Hypatia
400 - 460	Sun Zi
411 - 485	Proclus
476 - 550	Aryabhata I
598 - 670	Brahmagupta
735 - 804	Alcuin
790 - 850	al-Khwarizmi
940 - 1000	al-Quhi

about AD

1075 - 1160	Adelard
1114 - 1185	Bhaskara II
1114 - 1187	Gherard
1170 - 1250	Fibonacci
1195 - 1256	Sacrobosco
1220 - 1296	Campanus
1225 - 1260	Jordanus
1290 - 1349	Bradwardine
1323 - 1382	Oresme
1401 - 1464	Cusanus
1412 - 1492	Francesca
1445 - 1500	Chuquet
1445 - 1517	Pacioli
1465 - 1526	Ferro
1487 - 1567	Stifel
1492 - 1559	Ries
1500 - 1557	Tartaglia
1501 - 1576	Cardano
1522 - 1565	Ferrari
1526 - 1573	Bombelli
1540 - 1603	Viète
1560 - 1621	Harriot
1581 - 1638	Bachet
1588 - 1648	Mersenne
1596 - 1650	Descartes
1601 - 1665	Fermat
1611 - 1685	Pell
1616 - 1703	Wallis
1623 - 1662	Pascal, Blaise
1630 - 1677	Barrow
1643 - 1727	Newton
1646 - 1716	Leibniz
1656 - 1705	Bernoulli, Jacob

1661 - 1704	de L'Hôpital	1853 - 1928	Schönflies
1667 - 1748	Bernoulli, Johann	1858 - 1932	Peano
1690 - 1764	Goldbach	1859 - 1919	Hurwitz
1707 - 1783	Euler	1861 - 1941	Hensel
1717 - 1783	d'Alembert	1862 - 1943	Hilbert
1728 - 1777	Lambert	1863 - 1922	Thue
1734 - 1798	Waring	1864 - 1909	Minkowski
1736 - 1813	Lagrange	1865 - 1963	Hadamard
1741 - 1793	Wilson, John	1866 - 1962	Vallée Poussin
1749 - 1827	Laplace	1867 - 1938	Lehmer, D. N.
1752 - 1833	Legendre	1874 - 1954	Dickson
1768 - 1830	Fourier	1877 - 1947	Hardy
1776 - 1831	Germain	1877 - 1938	Landau
1777 - 1855	Gauß	1879 - 1967	Carmichael
1789 - 1857	Cauchy	1880 - 1975	Perron
1790 - 1868	Möbius	1880 - 1950	Fueter
1802 - 1829	Abel	1881 - 1977	Cunningham
1804 - 1851	Jacobi	1882 - 1935	Noether, Emmy
1805 - 1859	Dirichlet	1885 - 1977	Littlewood
1805 - 1865	Hamilton, W. R.	1887 - 1947	Hecke
1809 - 1882	Liouville	1887 - 1920	Ramanujan
1810 - 1893	Kummer	1888 - 1972	Mordell
1811 - 1832	Galois	1891 - 1983	Vinogradov
1814 - 1894	Catalan	1896 - 1981	Siegel
1821 - 1894	Chebyshev	1898 - 1962	Artin
1822 - 1901	Hermite	1898 - 1979	Hasse
1823 - 1852	Eisenstein	1903 - 1996	van der Waerden
1823 - 1891	Kronecker	1905 - 1981	Mazur
1826 - 1866	Riemann	1905 - 1991	Lehmer, D. H.
1829 - 1891	Wolstenholme	1906 - 2005	Wright
1831 - 1916	Dedekind	1906 -	Lehmer, Emma
1837 - 1920	Bachmann	1907 - 1969	Davenport
1842 - 1891	Lucas	1913 - 1996	Erdős
1842 - 1913	Weber, Heinrich	1926 -	Serre
1845 - 1918	Cantor, Georg	1953 -	Wiles
1852 - 1939	Lindemann	1954 -	Faltings

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