Computing Straight Skeletons and Motorcycle Graphs: Theory and Practice

Stefan Huber

Berichte aus der Informatik

Stefan Huber

Computing Straight Skeletons and Motorcycle Graphs: Theory and Practice

Shaker Verlag
Aachen 2012

Bibliographic information published by the Deutsche Nationalbibliothek
The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.

Zugl.: Universität Salzburg, Diss., 2011

Printed in Germany.

ISBN 978-3-8440-0938-5
ISSN 0945-0807

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen
Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9
Internet: www.shaker.de • e-mail: info@shaker.de

Abstract

The straight skeleton is a geometric structure that is similar to generalized Voronoi diagrams. Straight skeletons were introduced to the field of computational geometry one and a half decades ago. Since then many industrial and academical applications emerged, such as the computation of mitered offset curves, automatic roof construction, solving fold-and-cut problems and the reconstruction of surfaces, to name the most prominent ones. However, there is a significant gap between the most efficient straight-skeleton algorithms and implementations, on one hand, and the best known lower runtime bounds on the other hand. The primary goal of this thesis is the development of an algorithm that is suitable for implementation and efficient in terms of time and space in order to make the advantages of straight skeletons available to real-world applications.

We start with investigations concerning upper and lower bounds on the number of so-called flip events that occur in the triangulation-based straight-skeleton algorithm by Aichholzer and Aurenhammer. In particular, we prove the existence of Steiner triangulations that are free of flip events. This result motivates a novel straight-skeleton algorithm for non-degenerate simple polygons that is based on the so-called motorcycle graph. In order to extend this algorithm to arbitrary planar straight-line graphs, we carefully generalize the motorcycle graph. This generalization leads to practical and theoretical applications: Firstly, we obtain an extension of the alternative characterization of straight skeletons by Cheng and Vigneron to planar straight-line graphs. Secondly, this characterization motivates a straight-skeleton algorithm that is based on 3D graphics hardware. Thirdly, the generalized motorcycle graph leads to a wavefront-type straight-skeleton algorithm for arbitrary planar straight-line graphs. Our algorithm is easy to implement, has a theoretical worst-case time complexity of $O(n^2 \log n)$ and operates in $O(n)$ space. Extensive runtime tests with our implementation Bone exhibit an actual runtime of $O(n \log n)$ on a database containing more than 13 500 datasets of different characteristics. In practice, this constitutes an improvement of a linear factor in time and space compared to the current state-of-the-art straight-skeleton code, which is shipped with the CGAL library. In particular, Bone performs up to 100 times faster than the current CGAL code on datasets with a few thousand

vertices, requires significant less memory and accepts more general input.

The underlying algorithm of Bone motivates the investigation of motorcycle graphs and their practical computation. We start with stochastic considerations of the average length of a motorcycles trace. The results obtained motivate a simple yet fast algorithm that employs geometric hashing. Runtime tests with our implementation Moca exhibit an $O(n \log n)$ runtime on the vast majority of our datasets. Finally, we revisit the geometric relation of straight skeletons and motorcycle graph. We present an algorithm that constructs planar straight-line graphs whose straight skeleton approximates any given motorcycle graph to an arbitrary precision. This algorithm finally leads to a P-completeness proof for straight skeletons of polygons with holes that is based on the P-completeness of motorcycle graphs.

Acknowledgments

This book is a revision of my doctoral thesis which was accepted in July 2011 at the computer science department at the university of Salzburg, Austria. The work on this thesis was supported by the Austrian Science Fund (FWF), project no. L367-N15. I would like to thank my co-workers and colleagues for great collaboration. In particular, I would like to thank my academic adviser Prof. Martin Held for his support in the last years and for being a guide during my studies and beyond.

Finally, but most important, I thank my parents Gabriele and Stefan, my brother Andreas, and Christina.

Stefan Huber
Salzburg, January 2012

Contents