

Efficient Infinite Elements for Exterior Acoustics

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Daniel Dreyer
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Berichte aus der Akustik

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Notation

Conventions

$\approx$	Denotes the approximate version of $\cdot$
$\hat{\cdot}$	Time dependent version of $\cdot$
$\bar{\cdot}$	Conjugate of $\cdot$
$\cdot^T$	Transpose of $\cdot$
$\cdot^*$	Adjoint or hermitian conjugate of $\cdot$
A	Boldface uppercase denotes matrices
b	Boldface lowercase denotes vectors
$f(x) = o(g(x))$	For $x \rightarrow \infty$ the ratio $f(x)/g(x)$ approaches zero
$f(x) = O(g(x))$	Of the order, i.e. $f(x)/g(x)$ is bounded for all x
$\Re, \Im$	Real part, Imaginary part

Symbols

α, β	Coefficients of Jacobi polynomials 5.2.6
a	Radial distance of the base from the coordinate origin in the Astley-Leis elements 5.2.1, also representative problem size 7.1
B	Radiating or scattering structure 2.1.2
c	Wave velocity 2.1.1
$\mathbb{C}$	Set of complex numbers 2.1.2
d	Spatial dimension 2.1.2
dof	Degree of freedom 6.3.3
D, A, N	Frequency-independent system matrices for interior and exterior acoustics 3.1.1

ε_V	Volumetric strain 2.1.1
$\mathcal{E}_{rel}$	Relative error 5.2.8
$flops$	Floating-point operations, any of the type $+$, $-$, $*$, and $/$, 4.1.1
$\mathcal{F}$	Multiplicative radial weight in the test functions of the Astley-Leis elements 5.2.1
Γ	Envelope, separating Ω in Ω_i and Ω_e 2.2.1
Γ_X	Artificial boundary for enforcing the radiation or Sommerfeld condition 2.1.2
$h_\nu^{(2)}(kr)$	Spherical Hankel function of ν^{th} order and second kind 2.1.2
h	Representative element length 3.1.1
$\mathbf{h}$	Vector of $\tilde{H}_i$ 3.1.1
$\tilde{H}_i$	i^{th} finite element shape function 3.1.1
$\mathcal{H}_D^n$	Sobolev space of functions satisfying Dirichlet boundary conditions and being square integrable up to and including n^{th} derivatives 3.1.1
i	Imaginary unit 2.1.2
κ	Bulk modulus 2.1.1
k	Wave number 2.1.2
ka	Non-dimensional wave number 7
μ	Restart parameter of GMRES(μ) 4.2.3, and phase term in the Astley-Leis elements 5.2.1
m	Radial approximation order in the Astley-Leis elements 5.2.4
$\tilde{M}_\nu(v)$	ν^{th} radial geometric approximation function of type $1/r$ in the Astley-Leis elements 5.2.1
$\mathbf{M}$	Overall frequency-dependent system matrix 5.2.1
n	Surface normal vector 2.1.2, and dimension of matrices and vectors 3.1.1
np	Number of processors B.1.3
ω	Angular frequency 2.1.2
Ω	Fluid domain 2.1.2
Ω_i	Bounded inner fluid domain 2.2.1
Ω_e	Unbounded outer fluid domain 2.2.1

$\tilde{\Phi}_\nu$	ν^{th} approximation function in the Astley-Leis elements 5.2.1
p	Time-harmonic pressure 2.1.2, and approximation order of shape functions in the p -FEM 3.3
$\hat{p}$	Time dependent pressure 2.1.1
$\mathbf{p}$	Vector of pressure unknowns 3.1.1
$P_\nu(v)$	ν^{th} polynomial for radial approximation in the Astley-Leis elements 5.2.1
q	Test function 3.1.1
ρ	Density 2.1.1
$\mathbf{r}$	Right hand side 3.1.1
$\mathbb{R}$	Set of real numbers 2.1.1
$\mathbf{s}$	Vector of $\tilde{S}_i$ 3.1.1
s, t, v	Element-local coordinates 5.2.1
$\tilde{S}_i$	i^{th} $d - 1$ dimensional (face) finite element shape function 3.1.1
$\mathbf{S}$	Surface of the radiating or scattering structure B, so that $\partial\mathbf{B} = \mathbf{S}$ 2.1.2
t	Time 2.1.2
t_{asm}	Elapsed time for assembling the global system matrices 7.2.3
t_{norm}	Normalized elapsed time of solution phase 7.1.1
t_{sol}	Elapsed time of solution phase 7.2.3
u	Number of base nodes of an infinite element 5.2.1
$\mathbf{u}$	Infinitesimal displacement vector 2.1.1
v_n	Surface normal vibrations 3.1.1
$\mathbf{x}$	Vector of global coordinates x_i , $i = 1, \dots, d$ 2.1.1